

Project 1

This assignment follows the standard form for a project submission. You need to include an introduction, primary discussion, and summary. Include graphs, tables, and images, as necessary, to improve the clarity of your discussion. Your project needs to be both correct and well written. Communication remains a critical component of our modern, technological society. A few notes about format: you **MUST** use MS Word for your project and use Equation Editor for **all** mathematical symbols, e.g. $z(t) = \sin(t) + \frac{1}{\ln(t)}$.

If you have any questions about the requirements for this project, ask before you submit.

This project introduces approximations to Ordinary Differential Equations using numerical methods. You will program three numerical solvers: Euler's method, Improved Euler's method, and 4th order Runge-Kutta (RK4). **You are required to write your own numerical methods in either MATLAB or MS Excel.** You are not allowed to use numerical solvers written by anyone else.

Problem 1: Consider the following Initial Value Problem (IVP) where y is the dependent variable and t is the independent variable:

$$y' = \sin(t) * (1 - y) \text{ with } y(0) = y_0 \text{ and } t \geq 0$$

Note: the analytic solution for this IVP is:

$$y(t) = 1 + (y_0 - 1)e^{\cos(t)-1}$$

Part 1A: Approximate the solution to the IVP using Euler's method with the following conditions:

Initial condition $y_0 = -\frac{1}{2}$; time step $h = \frac{1}{16}$; and time interval $t \in [0,20]$

- + Derive the recursive formula for Euler's method applied to this IVP
- + Plot the Euler's method approximation
- + Plot the absolute error between the approximation and the exact solution using a semilog plot

Part 1B: Approximate the solution to the IVP using the Improved Euler's method with the following conditions: Initial condition $y_0 = -\frac{1}{2}$; time step $h = \frac{1}{16}$; and time interval $t \in [0,20]$

- + Derive the recursive formula for the Improved Euler's method applied to this IVP
- + Plot the Improved Euler's method approximation
- + Plot the absolute error between the approximation and the exact solution using a semilog plot

Part 1C: Approximate the solution to the IVP using the RK4 method with the following conditions:

Initial condition $y_0 = -\frac{1}{2}$; time step $h = \frac{1}{16}$; and time interval $t \in [0,20]$

- + Plot the RK4 method approximation
- + Plot the absolute error between the approximation and the exact solution using a semilog plot

Problem 2: Consider the following Initial Value Problem (IVP) where $y(t)$ is the dependent function:

$$y' = y - y^2 + 1.14 \cos(e^{t/2}) \text{ with } y(0) = y_0 \text{ and } t \geq 0$$

Part 2A: Approximate the solution to the IVP using the Improved Euler's method with the following conditions: Initial condition $y_0 = 1$; time steps $h = \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$; and time interval $t \in [0,20]$

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- + Plot the Improved Euler's method approximation for all 4 time steps
- + Discuss the results of these approximations

Part 2B: Approximate the solution to the IVP using the RK4 method with the following conditions:

Initial condition $y_0 = 1$; time steps $h = \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$; and time interval $t \in [0, 20]$

- + Plot the RK4 approximation for all 4 time steps
- + Discuss the results of these approximations

Grading Rubric

Projects provide you with an opportunity to improve your Mathematical skills as well as your communication. For this project you will need to correctly solve the problems and effectively communicate your ideas and solutions. This assignment will be evaluated across the areas of Validity, Readability, and Fluency.

Validity – Validity corresponds to the validity of your arguments. It addresses the extent to which your method is appropriate, your calculations are correct, and your analysis is accurate.

Readability – If your written work is not readable it cannot be assessed. Since the ability to communicate Mathematics is a focal point for this class, special attention will be paid to the readability of your work.

Fluency – Mathematics is a concise and precise language, and we wish to enhance your fluency. Therefore, part of every assessment will focus on your ability to incorporate correct, established notation and terminology into your written work

Evaluation criteria	Descriptive adjectives	Scoring
Validity	quality methods, correct solutions, proper conclusions, complete reasoning	40%
Readability	organization, presentation, format, clarity, effectiveness	35%
Fluency	proper notation, proper terminology, appropriate definitions, conciseness	25%

Course and Learning Objectives

This Project supports the following Course Objectives:

CO-2: Solve a variety of linear and nonlinear Ordinary Differential Equations (ODEs).

CO-5: Analyze the qualitative behavior of single variable, differential time models and their functional solutions.

CO-6: Synthesize several solution techniques to determine their appropriate application.